

The Rotating Black Hole in Renormalizable Quantum Gravity: The Three-Dimensional Hořava Gravity Case

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Abstract

Recently Hořava proposed a renormalizable quantum gravity, without the ghost problem, by abandoning Einstein's equal-footing treatment of space and time through the anisotropic scaling dimensions. Since then various interesting aspects, including the exact black hole solutions have been studied but no *rotating* black hole solutions have been found yet, except some limiting cases. In order to fill the gap, I consider a simpler three-dimensional set-up with $z = 2$ and obtain the exact rotating black hole solution. This solution has a ring curvature singularity inside the outer horizon, like the four-dimensional Kerr black hole in Einstein gravity, as well as a curvature singularity at the origin. The usual mass bound works also here but in a modified form. Moreover, it is shown that the conventional first law of thermodynamics with the usual Hawking temperature and chemical potential does not work, which seems to be the genuine effect of Lorentz-violating gravity due to lack of the absolute horizon.

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I. INTRODUCTION

Recently Hořava proposed a renormalizable gravity theory, without the ghost (i.e., unitarity) problem, which reduces to Einstein gravity in IR but with improved UV behaviors, by abandoning Einstein's equal-footing treatment of space and time through the anisotropic scaling dimensions, $[t] = -1, [\mathbf{x}] = -z$ with the dynamical critical exponents ($z > 1$) [1]. Since then various aspects have been studied, in particular several exact black hole solutions have been found [2–10]. But no *rotating* black hole solutions have been found yet, except some limiting cases [11] and so there have been some gap in Hořava gravity for describing our real black holes in the sky, which can be even nearly extremal, for example, $c_\ell J/GM^2 > 0.98$ in GRS 1915+105 [12] for the speed of light c_ℓ .

In order to fill the gap, in this paper I consider the three-dimensional set-up with $z = 2$, instead of studying the more challenging four-dimensional Hořava gravity with $z = 3$. By solving the three coupled non-linear equations for the three-dimensional $z = 2$ Hořava gravity with the general axisymmetric metric ansatz, I obtain the exact rotating black hole solution and study its physical properties. This solution has a ring curvature singularity inside the outer horizon, like the four-dimensional Kerr black hole in Einstein gravity, as well as a curvature singularity at the origin. The usual mass bound works also here but in a modified form. Moreover, it is shown that the conventional first law of thermodynamics with the usual Hawking temperature and chemical potential does not work, which seems to be the genuine effect of Lorentz-violating gravity due to lack of the absolute horizon.

II. THE ROTATING BLACK HOLE IN THREE-DIMENSIONAL HOŘAVA GRAVITY

Using the ADM decomposition of the metric

$$ds^2 = -N^2 c_\ell^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt) \quad (1)$$

the three-dimensional renormalizable action with $z = 2$ [13, 14], up to surface terms, is given by ¹

$$I = \frac{1}{\kappa} \int dt d^2x \sqrt{g} N (K_{ij} K_{ij} - \lambda K^2 + \xi R + \alpha R^2 - 2\Lambda), \quad (2)$$

where $\kappa = 16\pi G_3$,

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad (3)$$

is the extrinsic curvature, R is the Ricci scalar of the Euclidean two-geometry, λ, ξ are the IR Lorentz-violating parameters, and Λ is the cosmological constant. Note that in two-spatial

¹ In three-dimensional Lorentz-invariant space-time, it has been argued that the topologically massive gravity [15] may be renormalizable if suitable regularization is given [16]. But this action, which violates parity, exists only in the three dimensions and the renormalizability can not be generalized to four dimensions. Moreover, recently it has been clarified that the unitarity and renormalization are not compatible in three-dimensional Lorentz-invariant higher-curvature gravities, which preserves parity, for general coefficients of the higher-curvature terms [17], including the new massive gravity case [18].

dimensions all curvature invariants can be expressed by the Ricci scalar due to the identities, $R_{ijkl} = (g_{ik}g_{jl} - g_{il}g_{jk})R/2$, $R_{ij} = g_{ij}R/2$. Here, I do not consider the terms which depend on $a_i \equiv \partial_i N/N$ and $\nabla_j a_i$, which can change the IR as well as UV behaviors a lot from that of (2). Moreover, I do not consider the term of $\nabla^2 R$ [13] either since the qualitative structure of the solutions I will get is expected to be similar, as in the four dimensions [8].

Let me consider now an axially symmetric solution with the metric ansatz (I adopt the convention of $c_\ell \equiv 1$, hereafter)

$$ds^2 = -N^2(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 (d\phi + N^\phi(r)dt)^2. \quad (4)$$

Note that there is no angle (ϕ) dependance in the metric due to the circular symmetry in the two-dimensional space even with the rotation. By substituting the metric ansatz into the action (2), the resulting reduced Lagrangian, after angular integration, is given by

$$\mathcal{L} = \frac{2\pi}{\kappa} \frac{N}{\sqrt{f}} \left[\frac{fr^3 (N^{\phi'})^2}{2N^2} - \xi f' + \alpha \frac{f'^2}{r} - 2\Lambda r \right], \quad (5)$$

where the prime ($'$) denotes the derivative with respect to r . Note that there is only the ξ dependance but no λ dependance in the Lagrangian.

The equations of motions are

$$-\frac{fr^3 (N^{\phi'})^2}{2N^2} - \xi f' + \alpha \frac{f'^2}{r} - 2\Lambda r = 0, \quad (6)$$

$$\left(\frac{\sqrt{f}}{N} r^3 N^{\phi'} \right)' = 0, \quad (7)$$

$$\left(\frac{N}{\sqrt{f}} \right)' \left(2\alpha \frac{f'}{r} - \xi \right) + 2\alpha \frac{N}{\sqrt{f}} \left(\frac{f''}{r} - \frac{f'}{r^2} \right) = 0 \quad (8)$$

by varying the functions N , N^ϕ , and f , respectively.

For arbitrary α , Λ and ξ , I obtain the general solution

$$\begin{aligned} f &= -\mathcal{M} + \frac{br^2}{2} \left[1 - \sqrt{a + \frac{c}{r^4}} + \sqrt{\frac{c}{r^4}} \ln \left(\sqrt{\frac{c}{ar^4}} + \sqrt{1 + \frac{c}{ar^4}} \right) \right], \\ \frac{N}{\sqrt{f}} &\equiv W = 1 - \ln \sqrt{1 + \frac{c}{ar^4}}, \\ N^\phi &= -\frac{\mathcal{J}}{2r^2} \left[2 - \ln \sqrt{1 + \frac{c}{ar^4}} - \sqrt{\frac{ar^4}{c}} \arctan \left(\sqrt{\frac{c}{ar^4}} \right) \right] \end{aligned} \quad (9)$$

with

$$a = 1 + \frac{8\alpha\Lambda}{\xi^2}, \quad b = \frac{\xi}{2\alpha}, \quad c = \frac{2\alpha\mathcal{J}^2}{\xi^2}. \quad (10)$$

Here, I have set $W(\infty) \equiv 1$, $N^\phi(\infty) \equiv 0$ by choosing the appropriate coordinate system, without loss of generality, but they can be conventionally kept as independent parameters for

the analysis of the mass and angular momentum of the solution. Note that the parameters a, c are restricted to zero or positive values, i.e., $a, c \geq 0$, or equivalently,

$$\frac{8\alpha\Lambda}{\xi^2} \geq -1 \quad (11)$$

and $\alpha \geq 0$, for the real-valued metric functions f , N , and N^ϕ .

For large r and small α , the solution expands as

$$\begin{aligned} f &= -\frac{\Lambda}{\xi} r^2 \left(1 - \frac{2\alpha\Lambda}{\xi^2}\right) - \mathcal{M} + \frac{\mathcal{J}^2}{4r^2\xi} \left(1 - \frac{4\alpha\Lambda}{\xi^2}\right) - \frac{\alpha\mathcal{J}^4}{24\xi^3} \frac{1}{r^6} + \mathcal{O}(\alpha^2, r^{-10}), \\ W &= 1 - \frac{\alpha\mathcal{J}^2}{\xi^2} \frac{1}{r^4} + \mathcal{O}(\alpha^2, r^{-8}), \\ N^\phi &= -\frac{\mathcal{J}}{2r^2} + \frac{\alpha\mathcal{J}^3}{6\xi^2} \frac{1}{r^6} + \mathcal{O}(\alpha^2, r^{-10}). \end{aligned} \quad (12)$$

It is easy to check that, in the limit of $\alpha \rightarrow 0$, the solution reduces to the BTZ black hole solution (with $\xi = 1$) [19]

$$N_{BTZ}^2 = f_{BTZ} = -\frac{\Lambda}{\xi} r^2 - \mathcal{M} + \frac{\mathcal{J}^2}{4r^2\xi}, \quad N_{BTZ}^\phi = -\frac{\mathcal{J}}{2r^2}. \quad (13)$$

The non-vanishing curvature invariants are

$$R = -\frac{f'}{r} = -b \left(1 - \sqrt{a + \frac{c}{r^4}}\right), \quad (14)$$

$$\begin{aligned} K^{ij}K_{ij} &= \frac{r^2}{2W^2} \left(N^{\phi'}\right)^2 \\ &= \frac{\mathcal{J}^2}{2r^4} \frac{\left(\ln \sqrt{1 + \frac{c}{ar^4}}\right)^2}{\left(1 - \ln \sqrt{1 + \frac{c}{ar^4}}\right)^2} \end{aligned} \quad (15)$$

and (15) shows a ring curvature singularity when $W = 0$, i.e., at

$$r_{ring} = \left(\frac{c}{a(e^2 - 1)}\right)^{1/4} \approx \left(0.1565 \frac{c}{a}\right)^{1/4} \quad (16)$$

for the rotating solution, as well as a curvature singularity at $r = 0$ in both R and $K^{ij}K_{ij}$.

Note that the existence of a ring singularity is analogous to the four-dimensional Kerr black hole case but the singularity at $r = 0$ is not. For the BTZ black hole in Einstein gravity ($\alpha = 0$, $\xi = 1$), the curvature singularity at $r = 0$ in R is canceled by $K^{ij}K_{ij} - K^2$ and the remainders, which become the boundary terms in the action, in the (covariant) three curvature scalar $R^{(3)}$, resulting the finite value: $R^{(3)} = R + K^{ij}K_{ij} - K^2 - f'/r - f'' = 6\Lambda$. This means that the curvature singularity at $r = 0$ in R is the artifact of the time-foliation and not the physical singularity in Einstein gravity. But for the general solutions (9), the curvature singularities at $r = 0$ in (14) and (15) are covariant in the foliation preserving diffeomorphism such that they are physical singularities.

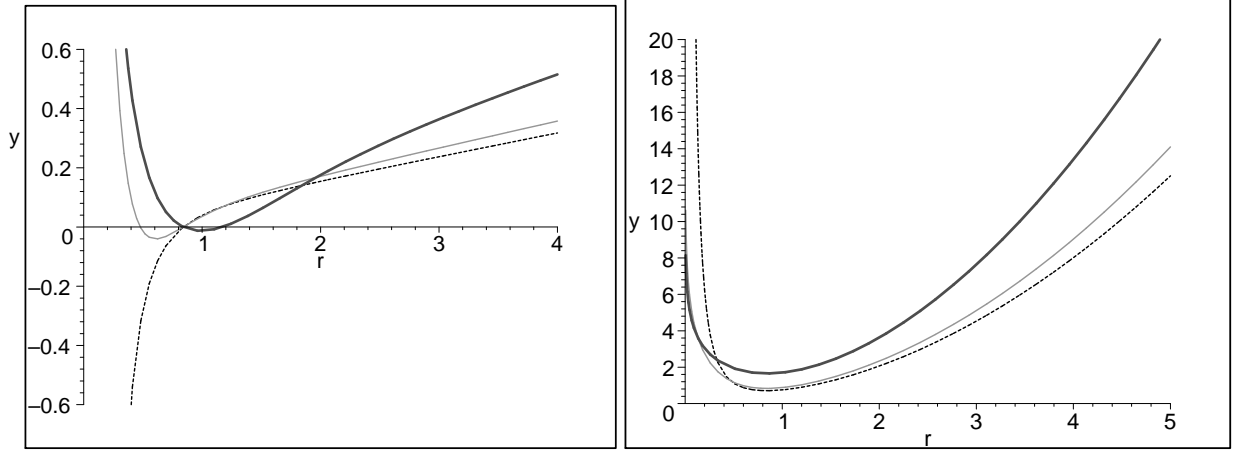


FIG. 1: Plots of T_+ (left) and \mathcal{M} (right) vs. r_+ for AdS space. The two solid curves represent the three-dimensional rotating Hořava black holes for different Lorentz-violating higher-derivative coupling $\alpha = 0.24, 0.1$ for the dark and bright curves, respectively, in comparison with the BTZ case ($\alpha = 0$) in the dotted curve. Here, I have considered $\xi = 1, \Lambda = -0.5, \mathcal{J} = 1$, and $\hbar \equiv 1$.

For asymptotically AdS, i.e., $\Lambda < 0$ ², the solution (9) has two horizons generally where f and N vanish simultaneously, i.e., the apparent and Killing horizons coincide, and the Hawking temperature for the outer horizon r_+ is given by

$$\begin{aligned} T_+ &= \frac{\hbar(Wf')|_{r_+}}{4\pi} \\ &= \frac{\hbar}{4\pi} b r_+ \left(1 - \sqrt{a + \frac{c}{r_+^4}} \right) \left(1 - \ln \sqrt{1 + \frac{c}{a r_+^4}} \right) \end{aligned} \quad (17)$$

from the regularity of the horizon in the Euclidean space-time, as usual. There is another Killing horizon when $W = 0$, i.e., $N = W\sqrt{f} = 0$ with $f \neq 0$, at $r = r_{ring}$ but this is not the event horizon since one can escape from (or reach to) the horizon in a finite time.³

In Fig.1 (left), the temperature T_+ vs. the outer horizon radius r_+ is plotted. For non-vanishing c , i.e., $\alpha, \mathcal{J} \neq 0$, there are two instances of the vanishing temperature:

(a) The first case is the usual extremal black hole limit, where the inner horizon r_+ meets the outer horizon r_+ at

$$r_+^* = \left(\frac{c}{1-a} \right)^{1/4} \quad (18)$$

and the integration constant

$$\mathcal{M} = \frac{b r_+^2}{2} \left[1 - \sqrt{a + \frac{c}{r_+^4}} + \sqrt{\frac{c}{r_+^4}} \ln \left(\sqrt{\frac{c}{a r_+^4}} + \sqrt{1 + \frac{c}{a r_+^4}} \right) \right] \quad (19)$$

² For asymptotically dS, i.e., $\Lambda > 0$, the solution (9) has the cosmological horizon as a generalization of KdS_3 in Einstein gravity [20]. And also, for asymptotically flat, i.e., $\Lambda = 0$, (9) has no horizon either. However, I will not consider these geometries here since the curvature singularities are then naked.

³ This may be compared with the non-commutative BTZ black hole case [21], where there is no coincidence point of the apparent and the Killing horizons and the smeared region is formed between them.

gets the minimum (Fig.1 (right)). This is the ground state in the usual black hole system and the outer horizon can not be smaller than r_+^* ; $T_+ < 0$ for $r_+ < r_+^*$ and this reflects a pathology of the region (for some related discussions, see Ref. [22]).

(b) The second case is the instance when $W|_{r_+} = 0$, i.e., when the outer horizon r_+ meets the ring curvature singularity r_{ring} of (16). If α is small enough so that $a > 1/e^2$, i.e., $-8\alpha\Lambda/\xi^2 < 1 - 1/e^2$, then this instance does not really occur since the outer horizon is always larger than the radius of the ring curvature singularity r_{ring} , i.e., $r_{ring} < r_+^* \leq r_+$. In this case the zero temperature is arrived when r_+ meets r_- at r_+^* before meets r_{ring} . This shows that the ring curvature singularity is safely protected by the outer horizon for the small α . However, if α is not so small so that $a \leq 1/e^2$, i.e., $-8\alpha\Lambda/\xi^2 \geq 1 - 1/e^2$, then there is the chance when r_+ meets r_{ring} from outside, i.e., $r_+ \geq r_{ring}$. But even in this case the ring singularity would not be naked since the zero temperature, i.e., the ground state is arrived by merging $r_+ \rightarrow r_{ring}$, and r_+ can not be smaller than r_{ring} : There is the ring singularity “on” the horizon, but this does not affect the outer region ($> r_+$) by the definition of the event horizon. In this case the second instance of the zero temperature is arrived before reaching the extremal black hole, except the case $a = 1/e^2$, where the extremal and ring singularity radius are degenerate, $r_+^* = r_{ring}$.

The ergo-region is defined by $g_{tt} = -N^2 + r^2(N^\phi)^2 \geq 0$ with its boundary at

$$r_{erg} = \frac{\sqrt{f}|W|}{|N^\phi|} \Big|_{r=r_{erg}} \quad (20)$$

and this region is outside of the outer horizon r_+ , i.e., $r_+ \leq r_{erg}$ since $f(r_{erg}) = g_{rr}(r_{erg}) \geq 0$ is required by (20). Here, the properties $W(r_{erg}) > 0$ from $r_{erg} \geq r_+ > r_{ring}$ and $N^\phi > 0$ are used.

Another peculiar property of the general solution is that there is the counter-rotating region inside the outer horizon r_+ (Fig.2). It is interesting to note that the turning point of N^ϕ , i.e., $N^{\phi'} = 0$ is at the location of the ring singularity $r = r_{ring}$ from $N^{\phi'} = W\mathcal{J}/r^3 = 0$ and the counter-rotating region starts at $r_{count} = (\eta c/a)^{1/4} (< r_{ring} < r_+)$ with $\eta \approx 0.0308$ which solves $N^\phi = 0$ ($\mathcal{J} \neq 0$).

III. THE UNUSUAL THERMODYNAMICS

The thermodynamics of Lorentz-violating black holes has not been well established yet⁴. In order to study this subject, I start by computing the conserved mass and angular momentum of the rotating black solution (9). To this ends, let me consider the variation of the total action $I_{total} = I + B$ with boundary terms B at space-like infinity such that the boundary variation $(\delta I)(\infty)$ is canceled by δB and there remain only the bulk terms in δI_{total} which vanish when the equations of motions hold. Then for the class of fields that approach our solution (9) at infinity, one finds

$$B = (t_2 - t_1)(-W(\infty)M + N^\phi(\infty)J), \quad (21)$$

which defines the canonical mass and angular momentum

$$M = \frac{2\pi\xi\sqrt{a}}{\kappa}\mathcal{M}, \quad J = \frac{2\pi\xi}{\kappa}\mathcal{J}, \quad (22)$$

⁴ But, see Ref. [23] for the black hole thermodynamics of non-rotating black holes.

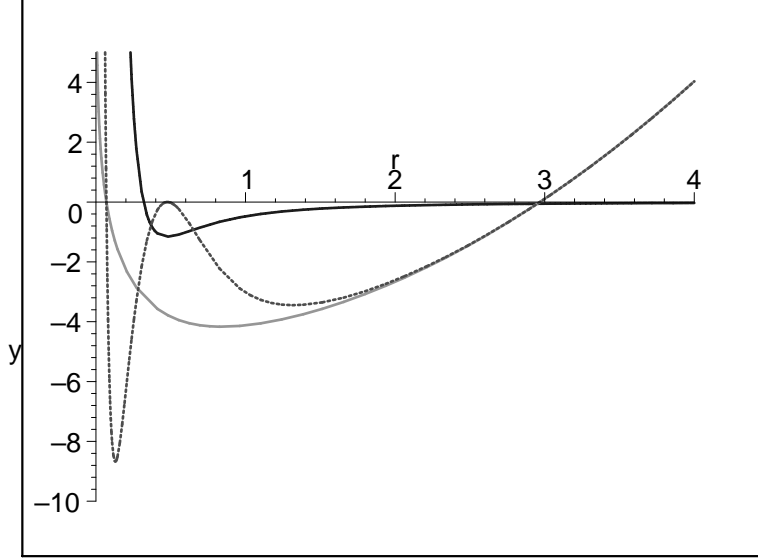


FIG. 2: Plots of $f(r)$ (bright solid), $N^2(r) = W^2 f$ (dotted), $N^\phi(r)$ (dark) curves for AdS space ($\xi = 1, \Lambda = -0.5, \mathcal{M} = 5, \mathcal{J} = 1, \alpha = 0.1$). In addition to the two horizons r_-, r_+ which are solutions of $f = 0$, $N^2 = 0$, simultaneously, N^2 has one more additional solution of $N^2 = W^2 f = 0$ ($f \neq 0$) at r_{ring} where the ring singularity is located, between r_- and r_+ . There is also the counter-rotation ($N^\phi > 0$) for $r < r_{count}$ and the turning point ($N^{\phi'} = 0$) is at r_{ring} .

as the conjugates to the asymptotic displacements $N(\infty)$ and $N^\phi(\infty)$, respectively, when kept as independent parameters.

In order that the curvature singularities are not naked, i.e., satisfying the cosmic censorship, one needs the mass bound condition

$$M \geq \chi(x) J \sqrt{-\Lambda} / |\xi| \quad (23)$$

with the monotonic function

$$\chi(x) = \sqrt{x^2 - 1} \ln \left(\frac{1}{\sqrt{x^2 - 1}} + \frac{1}{\sqrt{1 - x^{-2}}} \right) \quad (24)$$

which can vary in $[0, 1]$ as $x^2 \equiv \xi^2 / (-8\Lambda\alpha)$ varies in $[1, \infty]$. In the BTZ limit, $x^2 = \infty$, $\xi = 1$, one has the usual mass bound ($\chi = 1$)

$$M \geq J \sqrt{-\Lambda}, \quad (25)$$

but even for the other more general classes of $1 \leq \chi(x) < \infty$ so that $0 \leq \chi < 1$, the mass bound still works for each theory parameterized by x , but in a modified form.

Now in order to study the first law of black hole thermodynamics, let me consider the variation of the mass M as a function of J and r_+ ,

$$dM = A dJ + B dr_+ \quad (26)$$

with

$$\begin{aligned} A &= \frac{\kappa J}{4\pi\xi^2} \sqrt{\frac{a}{c}} \ln \left(\sqrt{\frac{c}{ar_+^4}} + \sqrt{1 + \frac{c}{ar_+^4}} \right), \\ B &= \frac{\pi\xi^2}{\kappa\alpha} r_+ \sqrt{a} \left(1 - \sqrt{a + \frac{c}{r_+^4}} \right). \end{aligned} \quad (27)$$

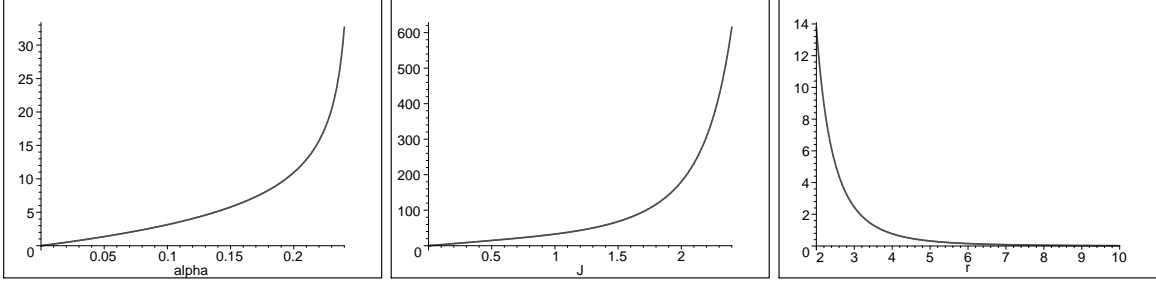


FIG. 3: Plots of $\partial_J \partial_{r_+} S - \partial_{r_+} \partial_J S$ vs. α (left), \mathcal{J} (middle) and r_+ (right) for AdS space ($\xi = 1, \Lambda = -0.5, \mathcal{J} = 1, \kappa = 1, \alpha = 0.24$ (middle, right), $r_+ = 2$ (left, middle)). The infinite barriers in the left and middle curves are due to $\alpha \leq -\xi^2/8\Lambda$ (11) and $M \leq \chi(x)J\sqrt{-\Lambda}/|\xi|$ (23).

Then, in order to see whether the first law of thermodynamics in the conventional form

$$dM = T_+ dS + \Omega_+ dJ \quad (28)$$

works with the usual Hawking temperature T_+ of (17) and the chemical potential $\Omega_+ = -N^\phi|_+$, let me define the black hole entropy function S with

$$dS \equiv \partial_{r_+} S dr_+ + \partial_J S dJ \quad (29)$$

as a function of r_+ and J . Then, from (26), (27), and (28), one can find

$$\partial_{r_+} S = \frac{B}{T_+}, \quad \partial_J S = \frac{\alpha \kappa^2 J}{\pi^2 \xi^4 T_+} \left(A - \Omega_+ \left(\frac{2\pi}{\kappa} \right)^2 \frac{\xi^3}{\alpha J} \right) \quad (30)$$

but $\partial_J \partial_{r_+} S - \partial_{r_+} \partial_J S \neq 0$, for arbitrary non-vanishing α , \mathcal{J} , and finite r_+ . The lengthy result for the non-integrability is not so impressive to be shown here but, in order to grasp how the Lorentz violation and the angular momentum affect the non-integrability, I show its leading term

$$\partial_J \partial_{r_+} S - \partial_{r_+} \partial_J S = \frac{16\pi^2 \mathcal{J}}{\kappa r_+^4} \alpha + \mathcal{O}(\alpha^2). \quad (31)$$

and the full results in the numerical plots (Fig. 3). These results show that the entropy is not integrable by the non-relativistic higher curvature corrections ($\alpha \neq 0$) for the rotating and finite black holes. The infinite barriers at α and J are due to $\alpha \leq -\xi^2/8\Lambda$ (11) and $M \leq \chi(x)J\sqrt{-\Lambda}/|\xi|$ (23). This proves that the entropy can *not* be defined in the conventional form of the first law of thermodynamics with the usual Hawking temperature and chemical potential.

IV. DISCUSSION

In conclusion, I have obtained the rotating black hole solution in the three-dimensional Hořava gravity where the Lorentz symmetry is broken by the higher-spatial derivatives in UV. Here, it is remarkable that the existence of the rotating black hole does not depend much on the existence nor the momentum dependance of speed, i.e., no absolute speed limit, of

gravitons. Actually, in our case there would be no graviton mode at the linear perturbation⁵ from the similar analysis in four-dimensional Hořava gravity since the calculation is not sensitive to the dimensionality of space [5, 24]. The status of its full, non-linear analysis is still unclear and needs more elaborative works with some ingenious separation of the genuine constraints, which being left as a further work.

And I have also shown that the mass bound condition still works in the new solution, analogous to the mass bound $GM^2 \geq c_\ell J$ for Kerr black hole in Einstein gravity. However, I have shown that the first law of thermodynamics can not be written in the conventional form with the usual Hawking temperature and the chemical potential such that the entropy function can not be defined for the generically rotating and Lorentz-violating black holes. The existence of Hawking temperature implies the Hawking radiation and this can be proved quite generally without knowing much details of the solutions (see for example Ref. [25]).

So, we have the black holes which generate the Hawking radiation but without the black hole entropy. Actually the notion of “Hawking radiation without black hole entropy” has been studied in the context of analogue black holes [26], previously. In our case, this seems to be the genuine effect of Lorentz-violating gravity due to lack of the absolute horizon which can leak the information depending on the matter’s momentum scale. This may be compared with other Lorentz-violating black holes, called Lifshitz black holes, where the first law of thermodynamics does not hold for a generic member of a class of black holes [27]. Finally, the study of rotating black holes in four-dimensional Hořava gravity would be quite a challenging problem which needs to solve five coupled equations, instead of three in our case.

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⁵ This is in contrast to the extended Hořava gravity [13] or the projectable model [14] where there is one scalar graviton mode.

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